Generation of 3-Dimensional graph state with Josephson charge qubits

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On the basis of generations of 1-dimensional and 2-dimensional graph states, we generate a 3-dimensional $N^3 - qubit$ graph state based on the Josephson charge qubits. Since any two charge qubits can be selectively and effectively coupled by a common inductance, the controlled phase transform between any two-qubit can be performed. Accordingly, we can generate arbitrary multi-qubit graph states corresponding to arbitrary shape graph, which meet the expectations of various quantum information processing schemes. All the devices in the scheme are well within the current technology. It is a simple, scalable and feasible scheme for the generation of various graph states based on the Josephson charge qubits.

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Entanglement [1] can serve as basic ingredient in the course of quantum information processing. In achieving the task of quantum communication, the entanglement is a medium for transferring quantum information. Owing to entanglement, quantum computers have potentially superior computing power over their classical counterparts. Graph states [2, 3, 4] are a family of multi-qubit states. Many well-known states, such as Greenberger-Horne-Zeilinger (GHZ) [5] states and cluster states [6], can be generated from the graph states. In quantum error correcting codes [7, 8] and in one-way quantum computing [9, 10] , some of graph states are used as the resources. For every non-trivial graph state it is possible to construct three-setting Bell inequalities which are maximally violated only by this state [11, 12].

The concept of a graph is the basis of a graph state. A graph $G = (V, E)$ comprise two classes of elements, *i.e.*, vertices V and edges E. Each graph can be represented by a diagram in a plane, where each vertex is represented by a point and each edge E by an arc joining two not necessarily distinct vertices [3]. For the graph states, vertices V correspond to qubits of physical systems and edges represent interactions of qubits. The state vector $|\Psi\rangle = |+\rangle^{\otimes V} = ((|0\rangle + |1\rangle)/\sqrt{2})^{\otimes V}$ is referred to as the graph state vector of the empty graph. The state vector of the graph state containing edges is described as

$$\begin{aligned} |G\rangle &= \prod_{(i,j\in E)} U^{(i,j)}|\Psi\rangle \\ &= \prod_{(i,j\in E)} U^{(i,j)}((|0\rangle + |1\rangle)/\sqrt{2})^{\otimes V} \qquad (1) \end{aligned}$$

with $U^{(i,j)} = (I^{(i)} \otimes I^{(j)} + \sigma_z^{(i)} \otimes I^{(j)} + I^{(i)} \otimes \sigma_z^{(j)} - \sigma_z^{(i)} \otimes \sigma_z^{(j)})/2$ corresponding to a controlled phase-gate between qubits labeled i and j, described by Pauli matrices. As mentioned above, the graph states have the special characteristics and practical applications, so the preparation of the graph states has become the focus of research. Clark *et al.* [7] present a scheme that allows arbitrary graph states to be efficiently created in a linear quantum register via an auxiliary entangling bus. Benjamin *et al.* [13] present a scheme that creates graph states by simple three-level systems in separate cavities. Bodiya *et al.* [14] propose a scheme for efficient construction of graph states using realistic linear optics, imperfect photon source and single-photon detectors.

Recently, much attention has been attracted to the quantum computer, which works on the fundamental quantum mechanical principle. The quantum computers can solve some problems exponentially faster than the classical computers. For realizing quantum computing, some physical systems, such as nuclear magnetic resonance [15], trapped irons [16], cavity quantum electrodynamics (QED) [17], and optical systems [18] have been proposed. These systems have the advantage of high quantum coherence, but can't be integrated easily to form large-scale circuits. Because of large-scale integration and relatively high quantum coherence, Josephson charge qubit [19, 20, 21] and flux qubit [22, 23], which are based on the macroscopic quantum effects in low-capacitance Josephson junction circuits [24, 25], are the promising candidates for quantum computing. As is well known, the graph states are mainly applied to quantum computing. Accordingly, generation of the graph states by Josephson charge and flux qubit is of great importance. In this paper, we propose a scheme for the generation of the graph states using Josephson charge qubit. This scheme is simple and easily manipulated, because any two charge qubits can be selectively and effectively coupled by a common inductance. More manipulations can be realized before decoherence sets in. All of the devices in the scheme are well within the current technology. It is a simple, scalable and feasible scheme for the generation of various graph states based on the Josephson charge qubits.

The paper is organized as follows: Firstly, we introduce Josephson charge-qubit structure and Hamiltonian of the system. Secondly, explain how to implement the controlled phase-gate. Thirdly, illustrate the generation of the arbitrary multi-qubit graph states corresponding to arbitrary shape graph. Fourthly, give necessary discussions for the feasibility of our scheme. Finally, the conclusions are given.

Since the earliest Josephson charge qubit scheme [19] was proposed, a series of improved schemes [20, 26] have been explored. Here, we concern the architecture of Josephson charge qubit in Ref. [26], which is the first efficient scalable

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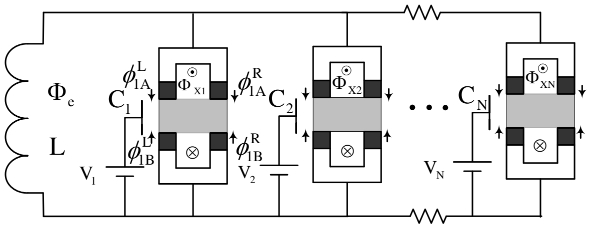

FIG. 1: Josephson charge-qubit structure. Each CBP is configure both in the charging regime $E_{ck} \gg E^0_{Jk}$ and at low temperatures $k_B T \ll E_{ck}$. Furthermore, the superconducting gap Δ is larger than E_{ck} so that quasiparticle tunneling is suppressed in the system.

quantum computing (QC) architecture. The Josephson charge qubits structure is shown in Fig.(1). It consists of N cooper-pair boxes (CPBs) coupled by a common superconducting inductance L. For the *k*th cooper-pair box, a superconducting island with charge $Q_k = 2en_k$ is weakly coupled by two symmetric direct current superconducting quantum interference devices (dc SQUIDs) and biased by an applied voltage through a gate capacitance C_k. Assume that the two symmetric dc SQUIDs are identical and all Josephson junctions in them have Josephson coupling energy E^0_{Jk} and capacitance C_{Jk}. The self-inductance effects of each SQUID loop is usually neglected because of the very small size ($1\mu m$) of the loop. Each SQUID pierced by a magnetic flux Φ_{Xk} provides an effective coupling energy $-E_{Jk}(\Phi_{Xk})\cos\phi_{kA(B)}$, with $E_{Jk}(\Phi_{Xk}) = 2E^0_{Jk}\cos(\pi\Phi_{Xk}/\Phi_0)$, and the flux quantum $\Phi_0 = h/2e$. The effective phase drop $\phi_{kA(B)}$, with subscript $A(B)$ labeling the SQUID above (below) the island, equals the average value $[\phi^L_{kA(B)} + \phi^R_{kA(B)}]/2$, of the phase drops across the two Josephson junctions in the dc SQUID, with superscript $L(R)$ denoting the left (right) Josephson junction.

For any given cooper-pair box, say i, when $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$, the inductance L connects only the ith cooper-pair box to form a superconducting loop. In the spin-$\frac{1}{2}$ representation, based on charge states $|0\rangle = |n_i\rangle$ and $|1\rangle = |n_{i+1}\rangle$, the reduced Hamiltonian of the system becomes [26]

$$H = \varepsilon_i(V_{Xi})\sigma_z^{(i)} - \overline{E}_{Ji}(\Phi_{Xi}, \Phi_e, L)\sigma_x^{(i)}, \qquad (2)$$

where $\varepsilon_i(V_{Xi})$ is controlled by the gate voltage V_{Xi}, while the intrabit coupling $\overline{E}_{Ji}(\Phi_{Xi}, \Phi_e, L)$ depends on inductance$L$, the applied external flux Φ_e through the common inductance and the local flux Φ_{Xi} through the two SQUID loops of the *i*th cooper-pair box. By controlling Φ_{Xk} and V_{Xk}, the operations of Pauli matrice $\sigma_z^{(i)}$ and $\sigma_x^{(i)}$ are achieved. Thus, any single-qubit operations are realized by utilizing the Eq. (1).

To manipulate many-qubit, say i and j, we configure $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$ and j. In the case, the inductance L is only shared by the cooper-pair boxes i and j to form superconducting loops. The Hamiltonian of the system can be reduced to [26, 27]

$$H = \sum_{k=i,j} [\varepsilon_k(V_{Xk})\sigma_z^{(k)} - \overline{E}_{Jk}\sigma_x^{(k)}] + \Pi_{ij}\sigma_x^{(i)}\sigma_x^{(j)}, \qquad (3)$$

where the interbit coupling Π_{ij} depends on both the external flux Φ_e through the inductance L, the local fluxes Φ_{Xi} and Φ_{Xj} through the SQUID loops. In Eq. (2), if we choose $V_{Xk} = (2n_k + 1)e/c_k$, the Hamiltonian of system can be reduced to

$$H = -\overline{E}_{Ji}\sigma_x^{(i)} - \overline{E}_{Jj}\sigma_x^{(j)} + \Pi_{ij}\sigma_x^{(i)}\sigma_x^{(j)}. \qquad (4)$$

For the simplicity of calculation, we assume $\overline{E}_{Ji} = \overline{E}_{Jj} = \Pi_{ij} = \frac{-\pi\hbar}{4\tau}$($\tau$ is a given period of time), which can be obtained by suitably choosing parameters. Thus Eq.(3) becomes

$$H = \frac{-\pi\hbar}{4\tau}(-\sigma_x^{(i)} - \sigma_x^{(j)} + \sigma_x^{(i)}\sigma_x^{(j)}). \qquad (5)$$

Below, we discuss problems on the basis $\{|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$. According to Hamiltonian H of Eq. (5), we can obtain the following evolutions:

$$|++\rangle_{ij} \rightarrow e^{-i\pi t/4\tau}|++\rangle_{ij}, \qquad (6a)$$

$$|+-\rangle_{ij} \rightarrow e^{-i\pi t/4\tau}|+-\rangle_{ij}, \qquad (6b)$$

$$|-+\rangle_{ij} \rightarrow e^{-i\pi t/4\tau}|-+\rangle_{ij}, \qquad (6c)$$

$$|--\rangle_{ij} \rightarrow e^{i3\pi t/4\tau}|--\rangle_{ij}. \qquad (6d)$$

If we choose $t = \tau$, which can be achieved by choosing switching time, and perform a single-qubit operation $U = e^{i\pi/4}$, we can obtain

$$|++\rangle_{ij} \rightarrow |++\rangle_{ij}, \qquad (7a)$$

$$|+-\rangle_{ij} \rightarrow |+-\rangle_{ij}, \qquad (7b)$$

$$|-+\rangle_{ij} \rightarrow |-+\rangle_{ij}, \qquad (7c)$$

$$|--\rangle_{ij} \rightarrow -|--\rangle_{ij}. \qquad (7d)$$

The Eq. (7) have actually realized the operation of a controlled phase gate. Any two charge qubits can be selectively and effectively coupled by a common inductance, so the controlled phase transform between any two-qubit is performed. It is very important for the following generation of arbitrary multi-qubit graph states corresponding to arbitrary shape graphs.

Under the basis of $\{|+\rangle, |-\rangle\}$, the state vector of the graph state containing edges is described as

$$|G\rangle = \prod_{(i,j\in E)} U'^{(i,j)}((|+\rangle + |-\rangle)/\sqrt{2})^{\otimes V}, \qquad (8)$$

where $U'^{(i,j)}$ is a controlled phase-gate for the basis of $\{|+\rangle, |-\rangle\}$. Our goal is generating a 3-dimensional $N^3 - qubit$ graph states corresponding to a 3-dimensional graph.

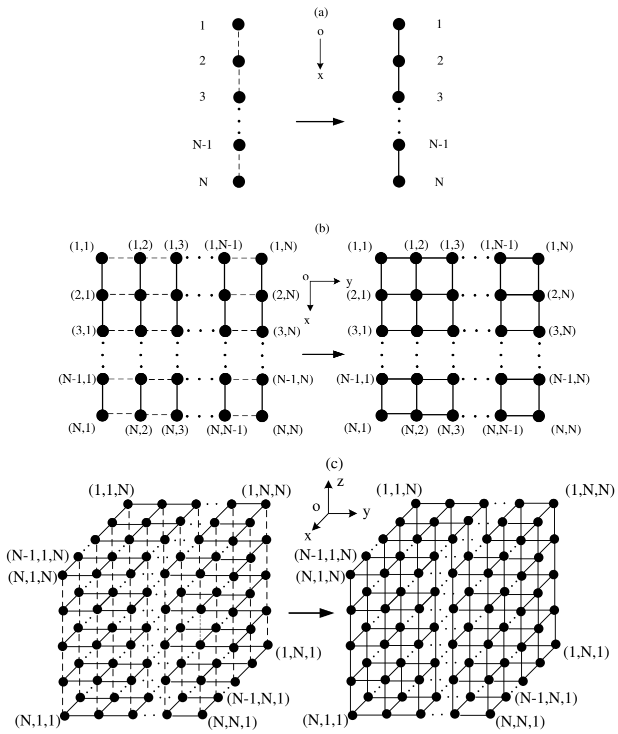

FIG. 2: The dashed lines in the figure denote that the interactions between qubits haven't taken place. The real lines denote the interactions between qubits have been completed. (a) Generation of N-qubit graph state from the graph state of empty graph to 1-dimensional graph state. (b) Generation of the graph state from 1-dimensional graph states to 2-dimensional. (c) Generation of the graph state from 2-dimensional graph states to 3-dimensional.

The work of generating 3-dimensional graph states divides into the following 3 steps.

Step 1: Firstly, we prepare all N charge qubits in the states of $|+\rangle$, which is the graph state of empty graph. Next, perform $N-1$ controlled phase transforms between adjacent charge qubits in x axis direction as shown in fig $2(a)$. Thus we obtain a 1-dimensional graph state corresponding to the right graph of the fig $2(a)$.

Step 2: Firstly, we prepare N graph states of 1-dimension, which is N-qubit graph state corresponding to the right graph in the fig $2(a)$. Next, on the basis of the left graph of the fig $2(b)$, perform $N(N-1)$ controlled phase transforms between adjacent charge qubits in y axis direction as shown in fig $2(b)$. Thus we obtain a 2-dimensional graph state corresponding to the right graph in the fig $2(b)$.

Step 3: Firstly, we prepare N graph states of 2-dimension, which is $N^2-qubit$ graph state corresponding to the right graph in the fig $2(b)$. Next, on the basis of the left graph of the fig $2(c)$, perform $N^2(N-1)$ controlled phase transforms between adjacent charge qubits in z axis direction as shown in fig $2(c)$. Thus we obtain a 3-dimensional graph state, that is a $N^3-qubit$ graph state corresponding to the right graph in the fig $2(c)$.

It is note to add that our scheme can generalize to generate arbitrary multi-qubit graph states corresponding to arbitrary shape graphs, which meet the expectations of various quantum information processing schemes.

Below, we briefly discuss the experimental feasibility of the current scheme. For the used charge qubit in our scheme, the typical experimental switching time $\tau^{(1)}$ during a single-bit operation is about $0.1ns$ [26]. The inductance L in our used proposal is about $30nH$, which is experimentally accessible. In the earlier design [20], the inductance L is about $3.6\mu H$, which is difficult to make at nanometer scales. Another improved design [24] greatly reduces the inductance L to $\sim 120nH$, which is about 4 times larger than the one used in our scheme. The fluctuations of voltage source and fluxes result in decoherence for all charge qubits. The gate voltage fluctuation plays the dominant role in producing decoherence. The estimated dephasing time is $\tau_4 \sim 10^{-4}s$ [24], which allow in principle 10^6 coherent single-bit manipulations. Owing to using the probe junction, the phase coherence time is only about $2ns$ [28, 29]. In this setup, background charge fluctuations and the probe-junction measurement may be two of the major factors in producing decoherences [26]. The charge fluctuations are principal only in the low-frequency region and can be reduced by the echo technique [28] and by controlling the gate voltage to the degeneracy point, but an effective technique for suppressing charge fluctuations still needs to be explored. According to discuss above, all of devices in our scheme are made by the current technology.

In summary, we have investigated a simple scheme for generating the graph states based on the Josephson charge qubit. Firstly, we generate a 1-dimensional $N-qubit$ graph state from a graph state corresponding to empty graph. Next, on the basis of the first step, generate a 2-dimensional $N^2-qubit$ graph state. Finally, on the basis of the second step, generate a 3-dimensional $N^3-qubit$ graph state. Since any two charge qubits can be selectively and effectively coupled by a common inductance, the controlled phase transform between any two-qubit is performed. Accordingly, we can generate arbitrary multi-qubit graph states corresponding to arbitrary shape graph, which meet the expectations of various quantum information processing schemes. The architecture of our proposal is made by present scalable microfabrication technique. More manipulations can be realized before decoherence sets in. All the devices in the scheme are well within the current technology. It is a simple, scalable and feasible scheme for the generation of various graph states based on the Josephson charge qubits.

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